

Higgs Inflation, Quantum Smearing and the Tensor to Scalar Ratio

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Abstract

In cosmic inflation driven by a scalar gauge singlet field with a tree level Higgs potential, the scalar to tensor ratio r is estimated to be larger than 0.036, provided the scalar spectral index $n_s \geq 0.96$. We discuss quantum smearing of these predictions arising from the inflaton couplings to other particles such as GUT scalars, and show that these corrections can significantly decrease r . However, for $n_s \geq 0.96$, we obtain $r \geq 0.02$ which can be tested by the Planck satellite.

In any realistic inflationary cosmology [1] the scalar inflaton field must couple to additional fields in order to implement the transition to a radiation dominated universe. These couplings, through quantum corrections (smearing), modify the tree level inflationary potential and therefore the corresponding predictions for the scalar spectral index n_s , tensor to scalar ratio r (measure of gravity waves), and the running of the spectral index $dn_s/d\ln k$. In Ref. [2], it was shown that quantum corrections of the chaotic inflationary potential, computed ala Coleman-Weinberg [3], and induced via couplings of the inflaton to other fields can significantly modify the tree level inflationary predictions. For instance, in the case of chaotic ϕ^2 inflation [4] supplemented by a Yukawa interaction, the tree level prediction of $n_s = 0.966$ and $r = 0.13$ is replaced by $0.93 \lesssim n_s \lesssim 0.966$ and $0.023 \lesssim r \lesssim 0.135$ [2]. A similar analysis has also been carried out for a non-supersymmetric hybrid inflationary model in Ref. [5], where inclusion of these quantum corrections is shown to allow even sub-Planckian values of the inflaton field, consistent with the red tilted spectral index exhibited in the WMAP 7 year (WMAP7) results [6]. The presence of a suitable Yukawa coupling, especially one involving right handed neutrinos, has one significant feature as far as realistic inflation model building is concerned. While enabling the transition from an inflaton to a radiation dominated universe, the decay products in this case contain right handed neutrinos whose subsequent out of equilibrium decay can give rise to the observed baryon asymmetry via leptogenesis (thermal [7] or non-thermal [8]). It was shown in [2, 5] that even though the radiative corrections during inflation may be sub-dominant, they can make sizeable corrections to the tree level predictions for n_s and r . Therefore, it is interesting to analyze the effects of these radiative corrections for various realistic inflationary models.

Motivated by the above observations, we investigate in this letter the impact of quantum corrections on the predictions of a Higgs inflation model in which a gauge singlet scalar ϕ plays the role of the inflaton field. For a tree level treatment of this model, see Refs. [9, 10, 11],

where it is shown that ϕ field has a trans-Planckian vacuum expectation value. Here we mainly consider the two important interaction terms of ϕ in the renormalizable Lagrangian: a quartic interaction between ϕ and a GUT symmetry breaking scalar boson Φ ($\sim \lambda_\Phi \phi^2 \Phi^2$), and a Yukawa interaction between ϕ and a right handed Majorana neutrino N ($\sim y \phi \bar{N} N$). It turns out that in order to obtain significant radiative corrections, both the Φ and N fields should be heavier than inflaton, and hence do not significantly contribute to reheating. Therefore, we keep the right handed neutrinos light enough to participate in reheating, while letting the GUT scalar boson be heavy enough to produce considerable radiative corrections. These radiative corrections are then shown to have a significant impact on the tree level predictions of n_s , r and $dn_s/\ln k$. A very precise measurement of n_s would be an extremely useful first step in the search for the correct inflation model. If n_s can be determined very precisely, say by the Planck satellite experiment, the different predictions for r which we report here would be an effective way to look for the quantum smearing effects that we have considered. In practice, we find that for $n_s \geq 0.96$, the scalar to tensor ratio $r \geq 0.02$, which can be tested by the Planck satellite.

We consider the Lagrangian density [2]

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial^\mu \phi_B \partial_\mu \phi_B + \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi + \frac{i}{2} \bar{N} \gamma^\mu \partial_\mu N \\ & + \frac{1}{2} m_B^2 \phi_B^2 - \frac{\lambda_B}{4} \phi_B^4 - \frac{1}{2} y_B \phi_B \bar{N} N + \frac{1}{2} \lambda_{\Phi_B}^2 \phi_B^2 \Phi^2 - \frac{a}{4} \Phi^4, \end{aligned} \quad (1)$$

where the subscript ‘B’ denotes bare quantities, and Φ represents the GUT symmetry breaking scalar boson. The GUT symmetry is broken when Φ acquires a non-zero vacuum expectation value (VEV) $\langle \Phi \rangle = (\lambda_\Phi/a)^{1/2} \phi$. In order to keep the discussion simple, we have introduced a single right handed neutrino N with Yukawa coupling y_B , and we ignore the bare mass terms for Φ and N . In a more realistic scenario, successful leptogenesis requires at least two right-handed neutrinos.

The inflationary potential including one loop corrections [3], in terms of renormalized quantities, can be written as

$$V = V_0 - \frac{1}{2} m^2 \phi^2 + \left(\frac{\lambda}{4} + \frac{\lambda_\Phi}{4a} \right) \phi^4 + A \phi^4 \left(\ln \left(\frac{\phi}{M} \right) + C \right), \quad (2)$$

where

$$A = \frac{1}{32\pi^2} (\mathcal{N} \lambda_\Phi^4 - 2y^4). \quad (3)$$

We have assumed $A \gg \lambda^2$ and $A \gg (m/\phi)^4$ so that the radiative correction from inflaton self couplings is suppressed [2]. Also, with $m_\Phi^2 \approx \lambda_\Phi^2 \phi^2 \gg H^2$ (H = Hubble constant), the ‘flat space’ quantum correction is a good approximation during inflation. Here \mathcal{N} is the dimensionality of the representation of the field Φ , V_0 is the vacuum energy density at the origin and C is a constant which can be determined from the minimization condition $V'(M) = 0$, where $M = \langle \phi \rangle$ denotes the VEV of ϕ at the minimum. Furthermore, if we require the potential to be zero at M , i.e. $V(M) = 0$, the value of V_0 is determined and the potential then takes the following form

$$V = \left(\frac{m^2 M^2}{4} \right) \left[1 - \left(\frac{\phi}{M} \right)^2 \right]^2 + A \phi^4 \left[\ln \left(\frac{\phi}{M} \right) - \frac{1}{4} \right] + \frac{A M^4}{4}, \quad (4)$$

where $V(\phi = 0) \equiv V_0 = \frac{m^2 M^2}{4} + \frac{A M^4}{4}$. The minimum of the above potential continues to lie at M as long as $m^2 + 2AM^2 > 0$ is satisfied. This condition is trivially met for the $A > 0$ case in which we are interested. The first term in the above potential is the usual tree level Higgs potential, whereas the second term is the Coleman-Weinberg Potential (CWP), and embodies the radiative corrections.

For the sake of completeness we first review some of the salient features and predictions of tree level Higgs inflation. The tree level Higgs potential can be written as [9, 10, 11]

$$V_{\text{tree}} = V_0 \left[1 - \left(\frac{\phi}{M} \right)^2 \right]^2, \quad (5)$$

with $V_0 = \frac{m^2 M^2}{4}$. As discussed in Refs. [9, 11], inflation may occur above or below the VEV M . For shorthand, we henceforth denote these regimes as the BV (below VEV) and AV (above VEV) solutions.

In the leading order approximation the slow-roll parameters are given as [1]

$$\epsilon = \frac{m_P^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = m_P^2 \left(\frac{V''}{V} \right), \quad \xi^2 = m_P^4 \left(\frac{V' V'''}{V^2} \right), \quad (6)$$

where $m_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The slow-roll approximation is valid as long as the conditions $\epsilon \ll 1$, $|\eta| \ll 1$ and $\xi^2 \ll 1$ hold. In this case the scalar spectral index n_s , the tensor-to-scalar ratio r , and the running of the spectral index $\frac{dn_s}{d \ln k}$ are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad (7)$$

$$r \simeq 16\epsilon, \quad (8)$$

$$\frac{dn_s}{d \ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \quad (9)$$

The number of e-foldings after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{2m_P^2} \int_{\phi_e}^{\phi_0} \frac{H(\phi) d\phi}{H'(\phi)}, \quad (10)$$

where ϕ_0 is the value of the field when the scale corresponding to k_0 exits the horizon, and ϕ_e is the value of the field at the end of inflation. The value of ϕ_e is given by the condition $2m_P^2(H'(\phi_e)/H(\phi_e))^2 = 1$, which can be calculated from the Hamilton-Jacobi equation [12]

$$[H'(\phi)]^2 - \frac{3}{2m_P^2} H^2(\phi) = -\frac{1}{2m_P^4} V(\phi). \quad (11)$$

Another expression of N_0 which explicitly depends on the thermal history of the universe is give by [13, 2]

$$N_0 \approx 65 + 2 \ln \left[\frac{V(\phi_0)^{1/4}}{m_P} \right] - \frac{4}{3\gamma_{reh}} \ln \left[\frac{V(\phi_e)^{1/4}}{m_P} \right] + \left(\frac{4}{3\gamma_{reh}} - 1 \right) \ln \left[\frac{\rho_{reh}^{1/4}}{m_P} \right], \quad (12)$$

$V_0^{1/4}(\text{GeV})$	$V(\phi_0)^{1/4}(\text{GeV})$	M/m_P	ϕ_0/m_P	ϕ_e/m_P	n_s	r	$m_\phi(\text{GeV})$	$T_{reh}(\text{GeV})$	N_0	$\frac{dn_s}{d\ln k}(10^{-4})$
1.3×10^{16}	1.3×10^{16}	12.7	2.03	11.7	0.944	0.020	1.5×10^{13}	5.1×10^7	54.9	-3.6
1.4×10^{16}	1.4×10^{16}	14.0	2.86	13.0	0.950	0.028	1.6×10^{13}	5.3×10^7	55.0	-4.3
1.5×10^{16}	1.4×10^{16}	14.8	3.42	13.8	0.953	0.033	1.7×10^{13}	5.3×10^7	55.0	-4.6
1.6×10^{16}	1.5×10^{16}	16.8	4.93	15.9	0.958	0.045	1.8×10^{13}	5.0×10^7	55.1	-5.2
1.7×10^{16}	1.6×10^{16}	18.1	5.94	17.1	0.960	0.051	1.8×10^{13}	4.8×10^7	55.2	-5.4
1.8×10^{16}	1.6×10^{16}	19.5	7.15	18.6	0.961	0.058	1.9×10^{13}	4.6×10^7	55.2	-5.7
1.9×10^{16}	1.7×10^{16}	21.2	8.61	20.3	0.962	0.065	1.9×10^{13}	4.2×10^7	55.2	-5.9
1.9×10^{16}	1.7×10^{16}	23.2	10.3	22.2	0.963	0.072	1.9×10^{13}	3.9×10^7	55.2	-6.0
2.0×10^{16}	1.8×10^{16}	25.4	12.3	24.4	0.964	0.078	1.9×10^{13}	3.6×10^7	55.2	-6.1
2.4×10^{16}	1.9×10^{16}	37.2	23.6	36.2	0.965	0.101	1.9×10^{13}	2.4×10^7	55.2	-6.4
2.8×10^{16}	1.9×10^{16}	50.0	36.0	49.0	0.965	0.112	1.8×10^{13}	1.7×10^7	55.2	-6.5
3.2×10^{16}	2.0×10^{16}	67.0	52.8	66.0	0.965	0.121	1.8×10^{13}	1.3×10^7	55.1	-6.6
3.7×10^{16}	2.0×10^{16}	89.5	75.2	88.5	0.964	0.127	1.8×10^{13}	9.2×10^6	55.1	-6.6

Table 1: Predicted values of various inflationary parameters using the tree level Higgs potential. Here we show only those values which fall inside the WMAP7 1σ bounds (see Fig. 1).

where ρ_{reh} is the energy density at the end of reheating, and $\gamma_{reh} = 2n/(n+2)$ for $V \propto \phi^n$ [14]. In particular, for ϕ^2 and ϕ^4 inflation $\gamma_{reh} = 1$ and $\gamma_{reh} = 4/3$ respectively. In the latter case N_0 is independent of ρ_{reh} . In our numerical calculations we take $\gamma_{reh} = 1$ because the quadratic component dominates near the minimum. Moreover, we assume quantum corrections to γ_{reh} to be negligible [2]. We can write $\rho_{reh} = \frac{\pi^2}{30} g_* T_{reh}^4$, where T_{reh} is the reheat temperature given by [15]

$$T_{reh} \simeq \left[\frac{30/g_*}{2\pi^3(1+w_{reh})(5-3w_{reh})} \right]^{1/4} \sqrt{\Gamma_\phi m_P}. \quad (13)$$

Here $w_{reh} = \gamma_{reh} - 1$ is the equation of state parameter for the dominant component during the reheating phase, $g_* = 106.75$ and $\Gamma_\phi \approx y^2 m_\phi / (8\pi)$ is the inflaton decay width.

The amplitude of the curvature perturbation is given by

$$\Delta_{\mathcal{R}} = \frac{1}{2\sqrt{3}\pi m_P^3} \frac{V^{3/2}}{|V'|}, \quad (14)$$

where $\Delta_{\mathcal{R}} = 4.93 \times 10^{-5}$ at $k_0 = 0.002 \text{ Mpc}^{-1}$ according to WMAP7 data [6]. Note that, for added precision, we include in our calculations the first order corrections in the slow-roll expansion for the quantities n_s , r , $dn_s/d\ln k$, and $\Delta_{\mathcal{R}}$ [16].

The predicted values of various parameters of the tree level Higgs inflation can be obtained by using Eqs. (5-14) above, and these are displayed in Table 1. As we see, the quantities M , ϕ_0 and ϕ_e carry trans-Planckian values but still the vacuum energy scale during observable inflation is well below m_P . This implies that the quantum gravity effects are relatively unimportant here. Moreover, very low values of the reheat temperature $T_{reh} \sim 10^7 \text{ GeV} \ll m_\phi \sim 10^{13} \text{ GeV}$ are realized with the Yukawa coupling $y \sim 10^{-6}$. Since we only consider the right handed Majorana neutrinos for reheating, we obtain negligible radiative corrections with their masses $m_N \leq m_\phi/2$ (or the Yukawa coupling $y \lesssim 10^{-6}$). Taking the number of e-foldings N_0 between 50 to 60, we obtain $0.945 \lesssim n_s \lesssim 0.967$ and $0.02 \lesssim r \lesssim 0.13$ within the WMAP7 1σ bounds as shown in Fig. 1. Furthermore, we only show the BV branch in Fig. 1 which lies largely within the

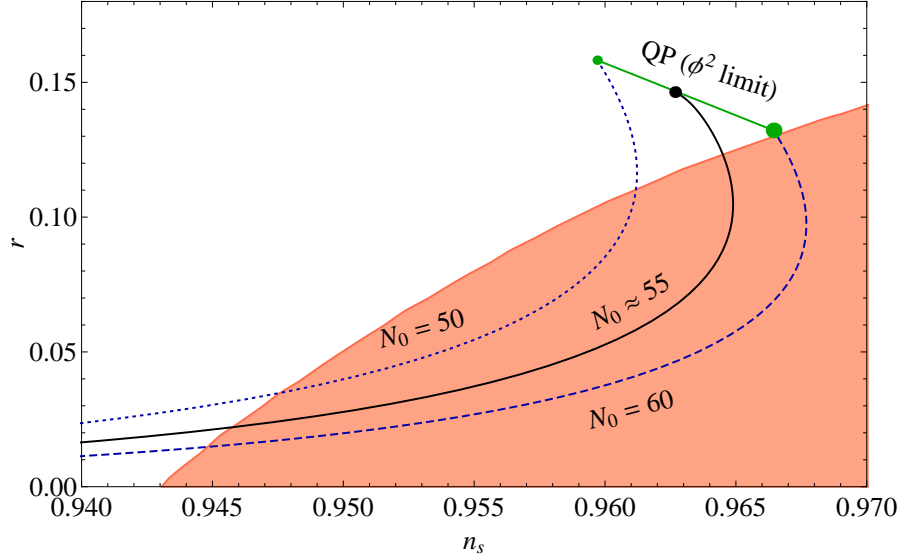


Figure 1: r vs. n_s for tree level Higgs inflation. The blue dotted, blue dashed and black solid curves correspond to number of e-foldings $N_0 = 50$, $N_0 = 60$ and N_0 given in Eq. (12), respectively. Small (big) green circles correspond to the Quadratic Potential (QP) with $N_0 = 50$ ($N_0 = 60$).

WMAP7 1σ bounds, whereas, the AV branch remains outside of the WMAP7 1σ bounds [11] at tree level. In this letter we mainly restrict our discussion to the WMAP7 1σ bounds.

The various limits of tree level Higgs inflation has been discussed in Ref. [11]. For example, for $\phi \ll M$, the BV branch approaches the effective potential of new inflation $V \approx V_0 \left(1 - 2\left(\frac{\phi}{M}\right)^2\right)$ in a region disfavored by WMAP7. On the other hand, for $\phi \gg M$, the AV branch approaches the effective potential of quartic inflation $V \approx \left(\frac{V_0}{M^4}\right) \phi^4$. As mentioned above this AV branch lies outside of the WMAP7 1σ bounds. Finally, near the VEV we obtain an effective potential of quadratic inflation $V \approx \frac{1}{2}m_\phi^2(\Delta\phi)^2$, where $\Delta\phi = M - \phi$ denotes the deviation of the field from the minimum and $m_\phi = \frac{2\sqrt{2V_0}}{M}$ is the inflaton mass. This chaotic inflationary model [4] predicts $m_\phi \simeq 2 \times 10^{13}$ GeV, $\Delta\phi_0 \simeq 2\sqrt{N_0}$, $n_s \simeq 1 - \frac{2}{N_0}$, and $r \simeq 4(1 - n_s)$, corresponding to $V(\phi_0) \simeq (2 \times 10^{16} \text{ GeV})^4$. In fact this is the region in which the two branches meet, i.e. both the BV and AV branches converge to quadratic inflation in the high- V_0 limit.

Let us now investigate the effect of including quantum smearing to the tree level Higgs inflation. In general we may consider two types of radiative corrections: fermionic radiative corrections (which arise from the Yukawa interaction that couples ϕ to fermions) and bosonic radiative corrections (which arise from the quartic interaction between ϕ and other bosons). As we see in Eq. (3), these radiative corrections appear in the potential with opposite signs and also exhibit different behavior in various inflationary predictions. These corrections become important around $|A| \sim 10^{-14}$ and if we consider the bosonic radiative corrections only, the

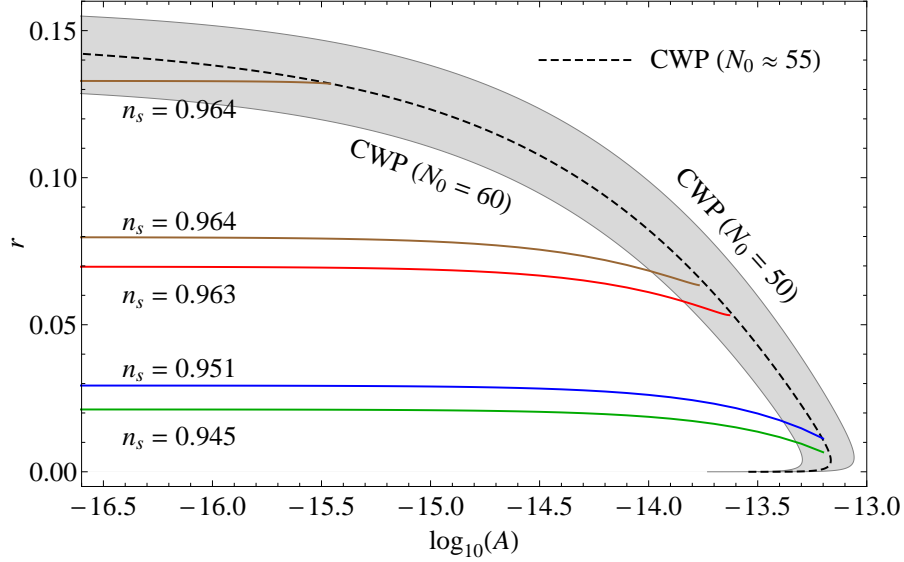


Figure 2: r vs. $\log_{10}(A)$ for the radiatively corrected Higgs potential for different values of n_s . Here the black dashed curve represents the predictions of Coleman-Weinberg Potential (CWP) with the number of e-foldings N_0 given in Eq. (12).

mass of the gauge scalar boson turns out to be of order the GUT scale $M_G \simeq \lambda_\Phi \langle \phi \rangle \sim 10^{16}$ GeV, with $\langle \phi \rangle \sim M_P$ and $\lambda_\Phi \sim 10^{-3}$. Here $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass. Similarly, in order to obtain sizable effects with the fermionic radiative corrections we need a Yukawa coupling y of order 10^{-3} . This, in turn, makes the fermion masses superheavy $\sim 10^{16}$ GeV, larger than the inflaton mass $m_\phi \sim 10^{13}$ GeV. Thus, these fermions are unable to participate in the reheating process and are also too heavy for seesaw physics. Consequently, we utilize the right handed Majorana neutrinos for reheating, which generates negligible radiative corrections with their masses $m_N \leq m_\phi/2$ and the Yukawa coupling $y \lesssim 10^{-6}$. In this regard, the Higgs inflationary model is different from the chaotic inflationary model where the same Yukawa interaction, responsible for the reheating, has also been shown to make sizable contributions, through radiative corrections, in modifications of the tree level predictions [2]. This is related to the fact that the Yukawa couplings in the chaotic inflation are not restricted by a non-zero inflaton VEV. In the following discussion, therefore, we only consider the effect of bosonic radiative corrections from the GUT field Φ .

The predictions of the bosonic radiatively corrected Higgs potential are shown in Fig. 2. The parameter A here quantifies the amount of radiative corrections. In order to explore the entire space of smearing, we show in Fig. 2 a plot between r and $\log_{10}(A)$ for the values of n_s which lie within the WMAP7 1σ bounds. As we increase the value of A , the tensor to scalar ratio r is observed to decrease. This is contrary to what we usually expect in the chaotic inflationary models where bosonic (fermionic) radiative corrections increase (reduce) the value of r . Actually, the BV branch of the tree level Higgs potential and the added bosonic radiative corrections have slopes with opposite signs, which reduces the magnitude of the total slope

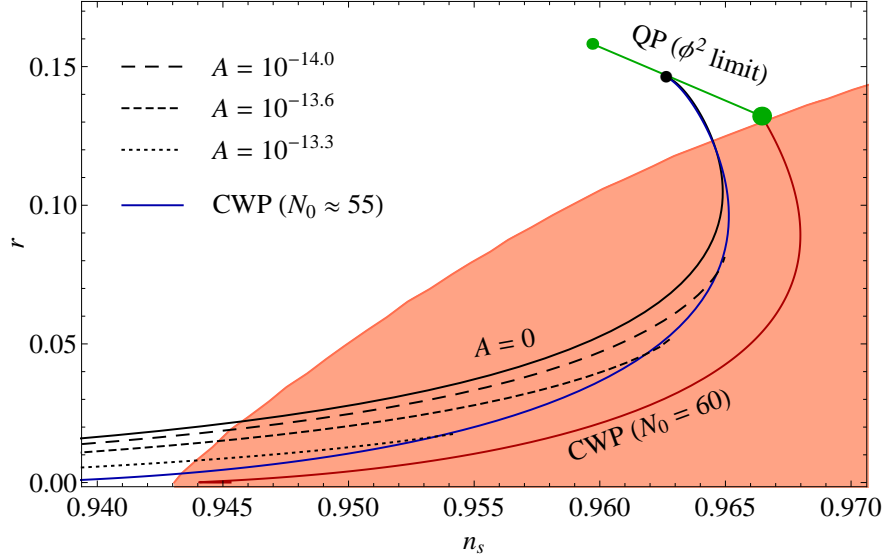


Figure 3: r vs. n_s for the tree level Higgs potential ($A = 0$) and the radiatively corrected Higgs potential ($A = 10^{-14.0}, 10^{-13.6}, 10^{-13.3}$). Blue and red curves represent the predictions of Coleman-Weinberg Potential (CWP) with the number of e-foldings $N_0 \approx 55$ using Eq. (12), and $N_0 = 60$ respectively. Small (big) green circles correspond to Quadratic Potential (QP) with $N_0 = 50$ ($N_0 = 60$).

of the potential and in turn decreases the value of r . In contrast, both the tree level chaotic inflationary potentials and their bosonic radiative corrections share slopes with same signs and this raises the value of r . A similar reduction in r has also been observed in Ref. [17] by adding higher order even powers of ϕ to the tree level Higgs potential. This reduction is prominent (insignificant) for small (large) values of the spectral index n_s and the tensor to scalar ratio r as a result of an increase (decrease) in the value of A . For example, with $n_s = 0.954$, $r \approx 0.02$ ($n_s = 0.964$, $r \approx 0.13$) the reduction in r is $\Delta r \approx 0.02$ ($\Delta r \approx 0$). Furthermore, the CWP curve provides an upper bound on A for a given value of n_s , as is clear from Fig. 2. For instance, the upper bound on A is $\sim 10^{-13.6}$ for the WMAP7 central value of spectral index $n_s = 0.963$. In the small A limit, all curves reproduce the standard tree level Higgs inflation predictions.

In Fig. 3, we fix the value of A and vary m between zero and its tree level value. We consider, as an example, three different values of the parameter $A = 10^{-14.0}$, $A = 10^{-13.6}$, and $A = 10^{-13.3}$. Each curve of radiatively corrected Higgs potential with constant value of A approaches the CWP prediction in the small m limit. As is clear from Fig. 3, the CWP curve is more contiguous to the central region of WMAP7 data in comparison to the tree level Higgs potential curve. Therefore, in Fig. 3 we also include a curve of CWP with $N_0 = 60$ which further extends the allowed space towards the central region of 1σ WMAP7 data. Moreover, both the tensor to scalar ratio r and the spectral index n_s are observed to decrease with an increase in A as explained above. The predictions of radiatively corrected Higgs potential, in fact, interpolate between the tree level Higgs potential and the CWP curves.

$V_0^{1/4}(\text{GeV})$	$V(\phi_0)^{1/4}(\text{GeV})$	A	M/m_P	ϕ_0/m_P	ϕ_e/m_P	r	$m_\phi(\text{GeV})$	$T_r(\text{GeV})$	$\frac{dn_s}{d\ln k}(10^{-4})$
1.91×10^{16}	1.72×10^{16}	$10^{-17.0}$	22.6	9.79	21.6	0.070	1.9×10^{13}	4.0×10^7	-6.0
1.91×10^{16}	1.72×10^{16}	$10^{-16.0}$	22.5	9.78	21.6	0.0670	1.9×10^{13}	4.0×10^7	-6.0
1.90×10^{16}	1.72×10^{16}	$10^{-15.0}$	22.4	9.72	21.5	0.069	1.9×10^{13}	4.1×10^7	-6.0
1.90×10^{16}	1.71×10^{16}	$10^{-14.8}$	22.4	9.67	21.4	0.068	1.9×10^{13}	4.1×10^7	-6.0
1.89×10^{16}	1.71×10^{16}	$10^{-14.6}$	22.3	9.62	21.3	0.067	1.9×10^{13}	4.1×10^7	-6.0
1.87×10^{16}	1.70×10^{16}	$10^{-14.4}$	22.1	9.53	21.2	0.066	1.9×10^{13}	4.2×10^7	-6.0
1.85×10^{16}	1.69×10^{16}	$10^{-14.2}$	22.0	9.44	21.0	0.064	1.9×10^{13}	4.2×10^7	-6.0
1.82×10^{16}	1.67×10^{16}	$10^{-14.0}$	21.7	9.36	20.8	0.061	1.9×10^{13}	4.3×10^7	-6.0
1.82×10^{16}	1.66×10^{16}	$10^{-13.9}$	21.7	9.35	20.7	0.060	1.9×10^{13}	4.3×10^7	-6.0
1.80×10^{16}	1.65×10^{16}	$10^{-13.8}$	21.6	9.40	20.7	0.058	1.9×10^{13}	4.4×10^7	-6.0
1.78×10^{16}	1.64×10^{16}	$10^{-13.8}$	21.7	9.48	20.7	0.057	1.9×10^{13}	4.4×10^7	-6.0
1.76×10^{16}	1.62×10^{16}	$10^{-13.7}$	22.0	9.91	21.0	0.055	2.0×10^{13}	4.4×10^7	-6.1
1.75×10^{16}	1.61×10^{16}	$10^{-13.7}$	22.4	10.4	21.5	0.054	2.0×10^{13}	4.3×10^7	-6.1
1.75×10^{16}	1.61×10^{16}	$10^{-13.6}$	23.3	11.3	22.3	0.053	2.0×10^{13}	4.1×10^7	-6.1

Table 2: Predicted values of various inflationary parameters using the radiatively corrected Higgs potential with $n_s = 0.963$ and $N_0 (\approx 55)$ given in Eq. (12).

To see the extent of quantum spread, we provide in Table 2 the predicted values of various parameters of this model corresponding to the the WMAP7 central value of the spectral index, $n_s = 0.963$. Most of the quantities exhibit a small variation from their respective tree level predictions. However, the the tensor to scalar ratio r experiences a significant shift. The tree level prediction of $r = 0.07$ acquires a total quantum spread of $0.05 \lesssim r \lesssim 0.07$ with $0 \lesssim A \lesssim 10^{-13.6}$.

The CWP prediction represents the maximal smearing of the radiatively corrected Higgs inflation results. This model has been studied extensively in the past [18, 19, 20] and recently [10, 11, 21]. The predicted values of various parameters of this model are displayed in Table 3 with the number of e-foldings $N_0 = 60$. Similar to tree level Higgs inflation, M , ϕ_0 and ϕ_e carry trans-Planckian values with vacuum energy scale well below m_P during observable inflation. We obtain $r \approx 0.03$ and $A \approx 3 \times 10^{-14}$ for the WMAP central value of the spectral index $n_s = 0.963$.

To summarize, we discuss the effect of including quantum corrections (smearing) in the tree level Higgs inflation. In contrast to chaotic inflation, a Yukawa interaction between inflaton and right handed neutrinos generates negligible radiative corrections owing to light ($\ll M_G$) neutrino masses. On the other hand, the bosonic radiative corrections, which are obtained from a quartic interaction between inflaton and a GUT symmetry breaking scalar boson, can yield noticeable effects. As a result of including these corrections, a reduction in the tensor to scalar ratio r is observed. The Coleman-Weinberg potential provides, in this case, the maximal quantum smearing limit. Using the WMAP7 central value of the spectral index $n_s = 0.963$, the tree level prediction of $r \approx 0.07$ is replaced by $0.05 \lesssim r \lesssim 0.07$. We emphasize that while working with high precision observations such as the current Planck satellite experiment we cannot ignore these radiative corrections in analyzing the predictions of various inflationary models.

$V_0^{1/4}(\text{GeV})$	$V(\phi_0)^{1/4}(\text{GeV})$	A	M/m_P	ϕ_0/m_P	ϕ_e/m_P	n_s	r	$m_\phi (\text{GeV})$	$\frac{dn_s}{d\ln k} (10^{-4})$
1.3×10^{16}	1.3×10^{16}	3.9×10^{-14}	17.0	6.24	16.1	0.960	0.020	1.6×10^{13}	-5.3
1.4×10^{16}	1.3×10^{16}	3.6×10^{-14}	18.1	7.10	17.2	0.961	0.024	1.7×10^{13}	-5.2
1.4×10^{16}	1.4×10^{16}	3.2×10^{-14}	19.4	8.10	18.5	0.962	0.028	1.7×10^{13}	-5.1
1.5×10^{16}	1.4×10^{16}	3.1×10^{-14}	20.2	8.67	19.2	0.963	0.030	1.7×10^{13}	-5.1
1.5×10^{16}	1.4×10^{16}	2.7×10^{-14}	21.7	9.95	20.8	0.964	0.035	1.8×10^{13}	-5.1
1.6×10^{16}	1.5×10^{16}	2.4×10^{-14}	23.5	11.4	22.6	0.965	0.040	1.8×10^{13}	-5.1
1.6×10^{16}	1.5×10^{16}	2.2×10^{-14}	24.5	12.3	23.6	0.965	0.043	1.8×10^{13}	-5.1
1.7×10^{16}	1.5×10^{16}	2.1×10^{-14}	25.3	13.0	24.4	0.965	0.045	1.8×10^{13}	-5.1
1.7×10^{16}	1.6×10^{16}	2.0×10^{-14}	26.1	13.7	25.2	0.966	0.047	1.8×10^{13}	-5.1
1.7×10^{16}	1.6×10^{16}	1.9×10^{-14}	26.7	14.2	25.7	0.966	0.049	1.8×10^{13}	-5.1
1.7×10^{16}	1.6×10^{16}	1.8×10^{-14}	27.3	14.7	26.3	0.966	0.050	1.8×10^{13}	-5.1
1.8×10^{16}	1.6×10^{16}	1.7×10^{-14}	28.2	15.5	27.3	0.966	0.052	1.8×10^{13}	-5.1
1.8×10^{16}	1.6×10^{16}	1.7×10^{-14}	28.5	15.8	27.6	0.966	0.053	1.8×10^{13}	-5.1
1.8×10^{16}	1.6×10^{16}	1.6×10^{-14}	28.9	16.1	27.9	0.966	0.054	1.8×10^{13}	-5.2
1.8×10^{16}	1.6×10^{16}	1.5×10^{-14}	29.9	17.0	28.9	0.967	0.056	1.8×10^{13}	-5.2
1.9×10^{16}	1.7×10^{16}	1.3×10^{-14}	32.0	19.0	31.0	0.967	0.060	1.8×10^{13}	-5.2
2.0×10^{16}	1.7×10^{16}	9.5×10^{-15}	37.8	24.4	36.9	0.968	0.070	1.8×10^{13}	-5.3
2.2×10^{16}	1.8×10^{16}	6.3×10^{-15}	45.5	31.7	44.5	0.968	0.080	1.8×10^{13}	-5.4
2.6×10^{16}	1.9×10^{16}	3.2×10^{-15}	62.2	48.0	61.2	0.968	0.094	1.7×10^{13}	-5.5
2.8×10^{16}	1.9×10^{16}	2.2×10^{-15}	74.1	59.7	73.1	0.968	0.100	1.7×10^{13}	-5.5
3.0×10^{16}	1.9×10^{16}	1.5×10^{-15}	90.4	75.8	89.4	0.968	0.106	1.7×10^{13}	-5.5
3.5×10^{16}	1.9×10^{16}	7.4×10^{-16}	124	110	123	0.967	0.113	1.6×10^{13}	-5.6
4.0×10^{16}	2.0×10^{16}	4.4×10^{-16}	159	144	158	0.967	0.117	1.6×10^{13}	-5.6
4.5×10^{16}	2.0×10^{16}	2.4×10^{-16}	212	197	211	0.967	0.121	1.6×10^{13}	-5.6
5.0×10^{16}	2.0×10^{16}	1.6×10^{-16}	256	241	255	0.967	0.123	1.6×10^{13}	-5.6
6.0×10^{16}	2.0×10^{16}	7.5×10^{-17}	374	358	373	0.967	0.126	1.6×10^{13}	-5.6
7.0×10^{16}	2.0×10^{16}	3.9×10^{-17}	519	504	518	0.967	0.128	1.6×10^{13}	-5.6
7.9×10^{16}	2.0×10^{16}	2.4×10^{-17}	655	640	654	0.967	0.129	1.6×10^{13}	-5.6

Table 3: Predicted values of various inflationary parameters using the Coleman-Weinberg Potential (CWP). Here we show only those values which fall inside the WMAP7 1σ bounds and satisfy $n_s \gtrsim 0.96$ and $N_0 = 60$.

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